Compute-Intensive Methods in AI: New Opportunities for Reasoning and Search

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Introduction

In recent years, we've seen substantial progress in propositional reasoning and search methods.

Boolean satisfiability testing:

1990: 100 variables / 200 clauses (constraints)

1998: 10,000 - 100,000 vars / 10^6 clauses

Novel applications:

e.g. in planning, software / circuit testing,
machine learning, and protein folding

Factors in Progress

a) new algorithms

e.g. stochastic methods

b) better implementations

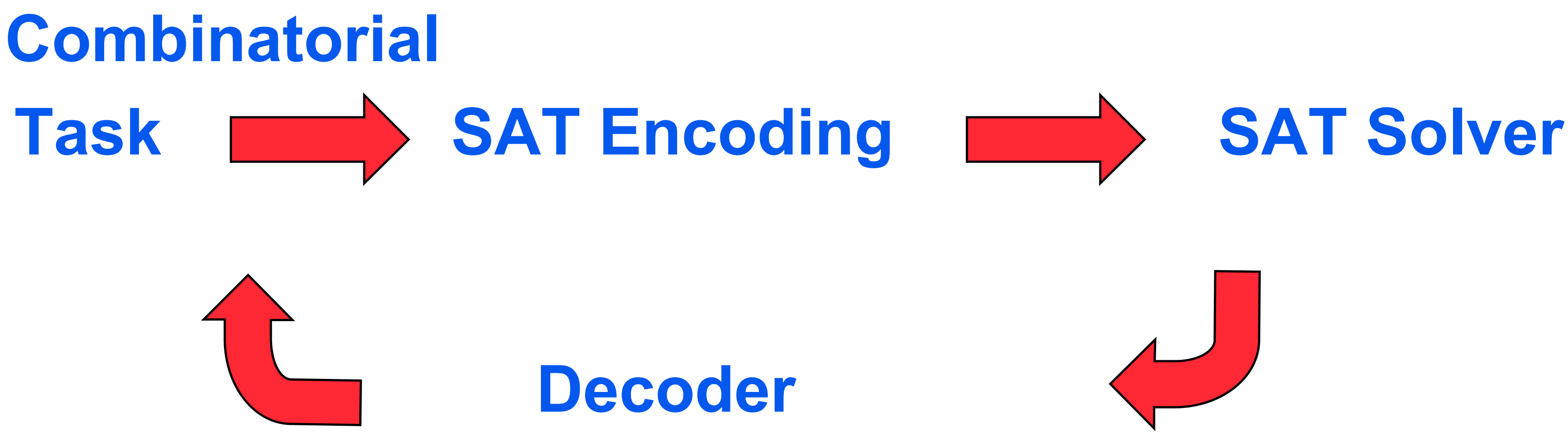
several competitions ---

Germany 91 / China 96 / DIMACS-93/97/98

c) faster hardware

Also, close interplay between theoretical, experimental, and applied work.

Applications: Methodology



Shift work to "encoding phase",

use fast, off-the-shelf SAT solver and tools.

Compare methodology to the use of

Linear / Integer Programming packages:

--- Emphasis is on mathematical modeling (e.g. using primal and dual formulations).

--- After modeling phase, problem is handed to a state-of-the-art solver.

Would specialized solver not be better?

Perhaps theoretically, but often not in practice

--- It's difficult to duplicate efforts put in designing fast solvers.

--- Encodings can compensate for much of the loss due to going to a uniform representation formalism (e.g. SAT, CSP, LP, or MIP).

Outline

I --- Example application: AI Planning
The SATPLAN system

II --- Current Themes in SAT Solvers
randomization / scalability

III --- Current Themes in SAT Encodings
declarative control knowledge

IV --- Conclusions

I. Example Application: Planning

Planning: **find a (partially) ordered set of actions that transform a given initial state to a specified goal state.**

- **in most general case, can cover most forms of problem solving**
- **special case of program synthesis**
- **scheduling: fixes set of actions, need to find optimal total ordering**

  - **planning problems typically highly non-linear, require combinatorial search**

Some Applications of Planning

Autonomous systems

Deep Space One Remote Agent (NASA)

mission planning

Softbots - software robots

- Internet agents, program assistants
- AI "characters" in games, entertainment

Synthesis, bug-finding (goal = undesirable state), …

Supply Chain Management --- *"just-in-time" manufacturing* (SAP, I2, PeopleSoft etc. $10 billion)

Proof planning in mathematical domains (Melis 1998)

State-space Planning

Find a sequence of operators that transform an initial state to a goal state

State = complete truth assignment to a set of variables (fluents)

Goal = partial truth assignment (set of states)

Operator = a partial function State → State

specified by three sets of variables:

precondition, add list, delete list

(STRIPS-style, Nilsson & Fikes 1971)

Abundance of Negative Complexity Results

I. Domain-independent planning: PSPACE-complete or worse

(Chapman 1987; Bylander 1991; Backstrom 1993)

II. Domain-dependent planning: NP-complete or worse

(Chenoweth 1991; Gupta and Nau 1992)

III. Approximate planning: NP-complete or worse

(Selman 1994)

Practice

Traditional domain-independent planners can generate plans of only a few steps.

- **Prodigy, Nonlin, UCPOP, ...**

Practical systems minimize or eliminate search by employing

- **complex search control rules, hand-tailored to the search engine and the particular search space**
  (Sacerdoti 1975, Slaney 1996, Bacchus 1996)
- **pre-compiling entire state-space to a reactive finite-state machine**
  (Agre & Chapman 1997, Williams & Nayak 1997)

Scaling remains problematic when state space is large or not well understood!

Progression

- **Planning as first-order theorem proving**
  (Green 1969)

  computationally infeasible

- **STRIPS (Fikes & Nilsson 1971)**

  very hard

- **Partial-order planning**
  (Tate 1977, McAllester 1991, Smith & Peot 1993)

  can be more efficient, but still hard
  (Minton, Bresina, & Drummond 1994)

- **Proposal: planning as propositional reasoning**

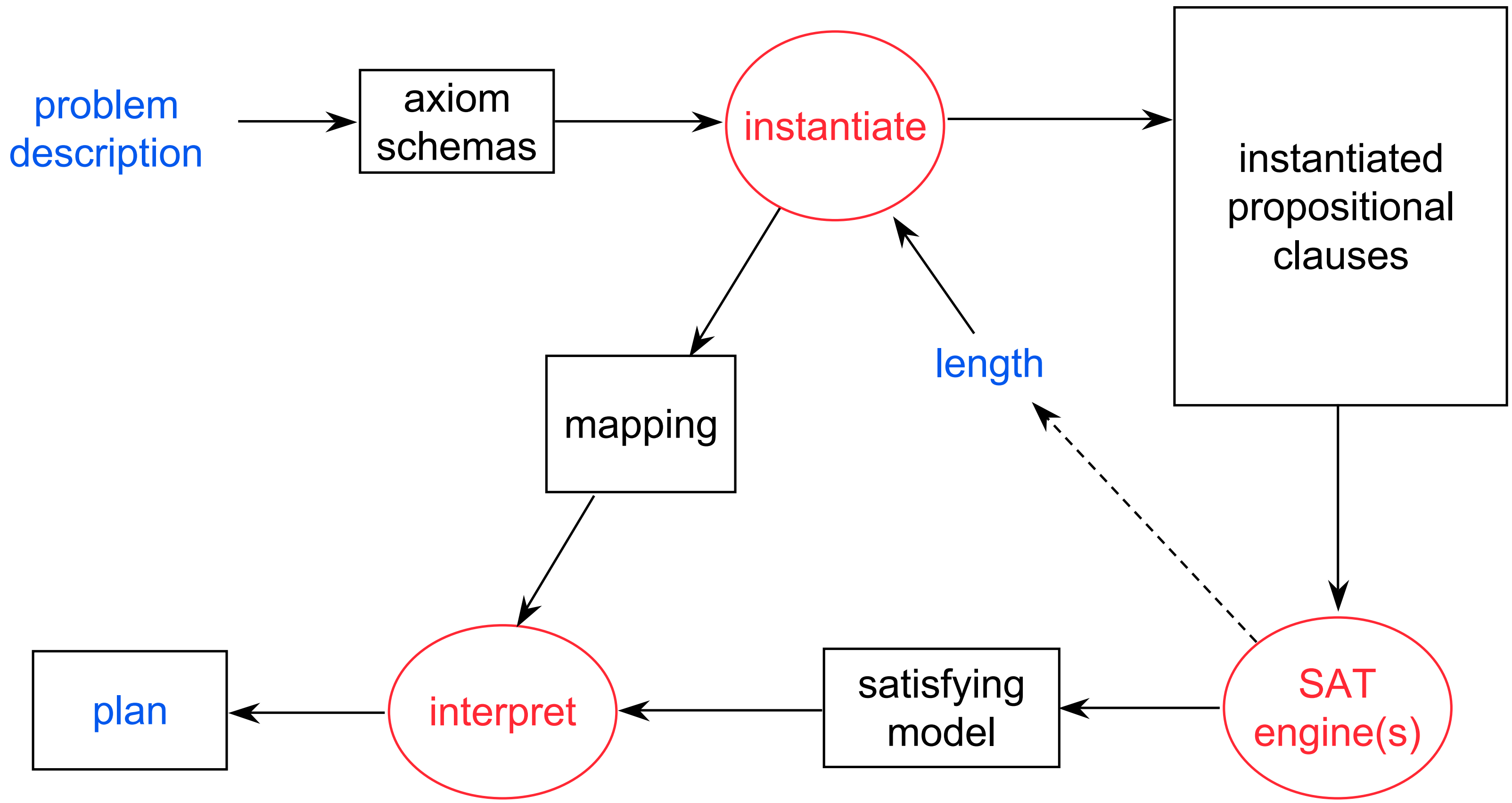
Approach

SAT encodings are designed so that plans correspond to satisfying assignments

Use recent efficient satisfiability procedures (systematic and stochastic) to solve

Evaluation performance on benchmark instances

SATPLAN



problem description
axiom schemas
instantiate
instantiated propositional clauses
mapping
length
SAT engine(s)
satisfying model
interpret
plan

SAT Encodings

Propositional CNF: no variables or quantifiers

Sets of clauses specified by axiom schemas

- fully instantiated before problem-solving

Discrete time, modeled by integers

- state predicates: indexed by time at which they hold
- action predicates: indexed by time at which action begins
  - each action takes 1 time step
  - many actions may occur at the same step

fly(Plane, City1, City2, i) ⊃ at(Plane, City2, i +1)

Solution to a Planning Problem

A solution is specified by any model (satisfying truth assignment) of the conjunction of the axioms describing the initial state, goal state, and operators

Easy to convert back to a STRIPS-style plan

Satisfiability Testing Procedures

Systematic, complete procedures

Depth-first backtrack search
(Davis, Putnam, & Loveland 1961)

unit propagation, shortest clause heuristic

State-of-the-art implementation: ntab
(Crawford & Auton 1997)

and many others! *See SATLIB 1998 / Hoos & Stutzle.*

Stochastic, incomplete procedures

GSAT (Selman et. al 1993)

Current fastest: Walksat (Selman & Kautz 1993)

greedy local search + noise to escape local minima

Walksat Procedure

Start with random initial assignment.

Pick a random unsatisfied clause.

Select and flip a variable from that clause:

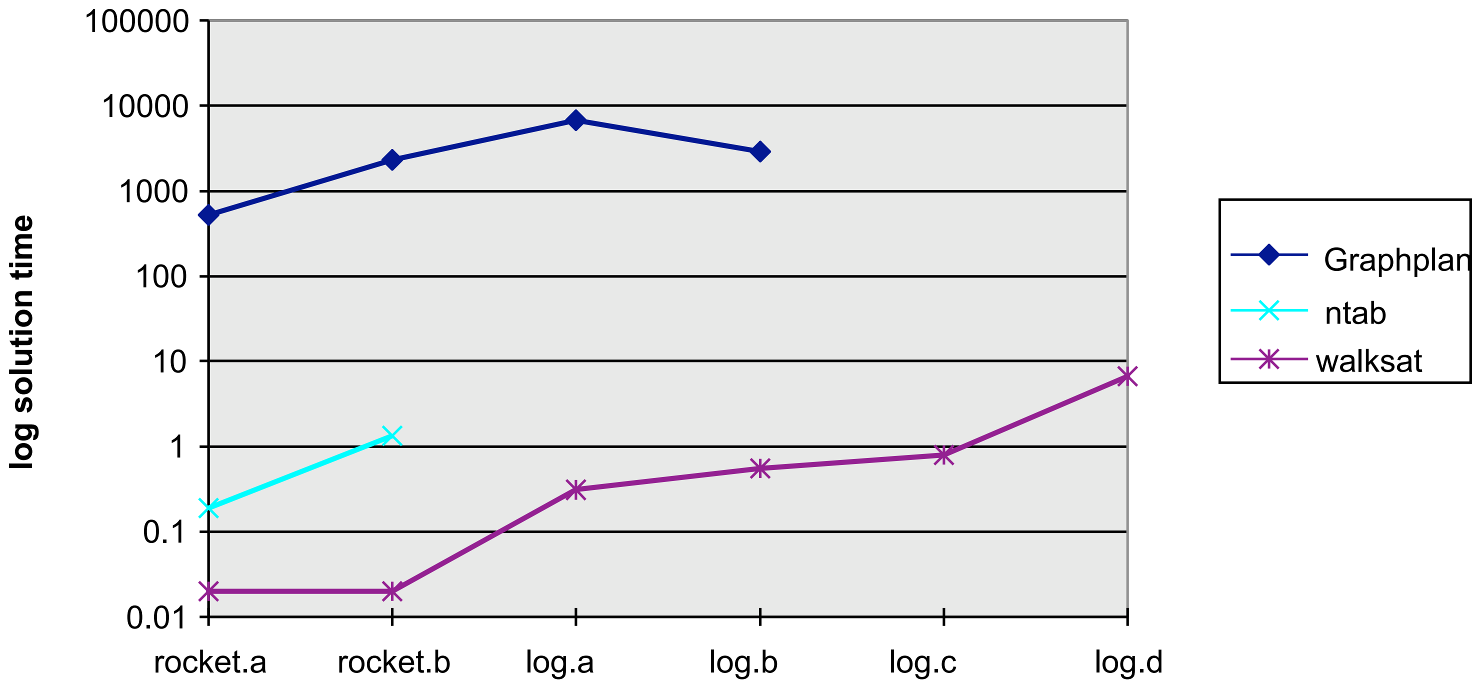
- With probability p, pick a random variable.
- With probability 1-p, pick greedily
  - a variable that minimizes the number of unsatisfied clauses

Repeat to predefined maximum number flips; if no solution found, restart.

Planning Benchmark Test Set

- Extension of Graphplan benchmark set
  - Graphplan faster than UCPOP (Weld 1992) and Prodigy (Carbonell 1992) on blocks world and rocket domains
- logistics - complex, highly-parallel transportation domain, ranging up to
  - 14 time slots, unlimited parallelism
  - 2,165 possible actions per time slot
  - optimal solutions containing 150 distinct actions
- *Problems of this size (10^18 configurations) not previously handled by any state-space planning system*

Solution of Logistics Problems



What SATPLAN Shows

A general propositional theorem prover can be competitive with specialized planning systems

- Surpise:
  - "Search direction" does not appear to matter. (Traditional planners generally backward chain from goal state.)
- Fast SAT engines
  - stochastic search - walksat
  - large SAT/CSP community sharing ideas and code
    - specialized engines can catch up, but by then, new general technique

II. Current Themes in Sat Solvers

SAT Solvers

Stochastic local search solvers (walksat)

- when they work, scale well
- cannot show unsat
- fail on certain domains
- must use very simple (fast) heuristics

Systematic solvers (Davis Putnam Loveland style)

- complete
- fail on (often different) domains
- might use more sophisticated (costly) heuristics
- often to scale badly

Can we combine best features of each approach?

Background

Combinatorial search methods often exhibit a remarkable variability in performance. It is common to observe significant differences between:

- different heuristics
- same heuristic on different instances
- different runs of same heuristic with different seeds (stochastic methods)

Preview of Strategy

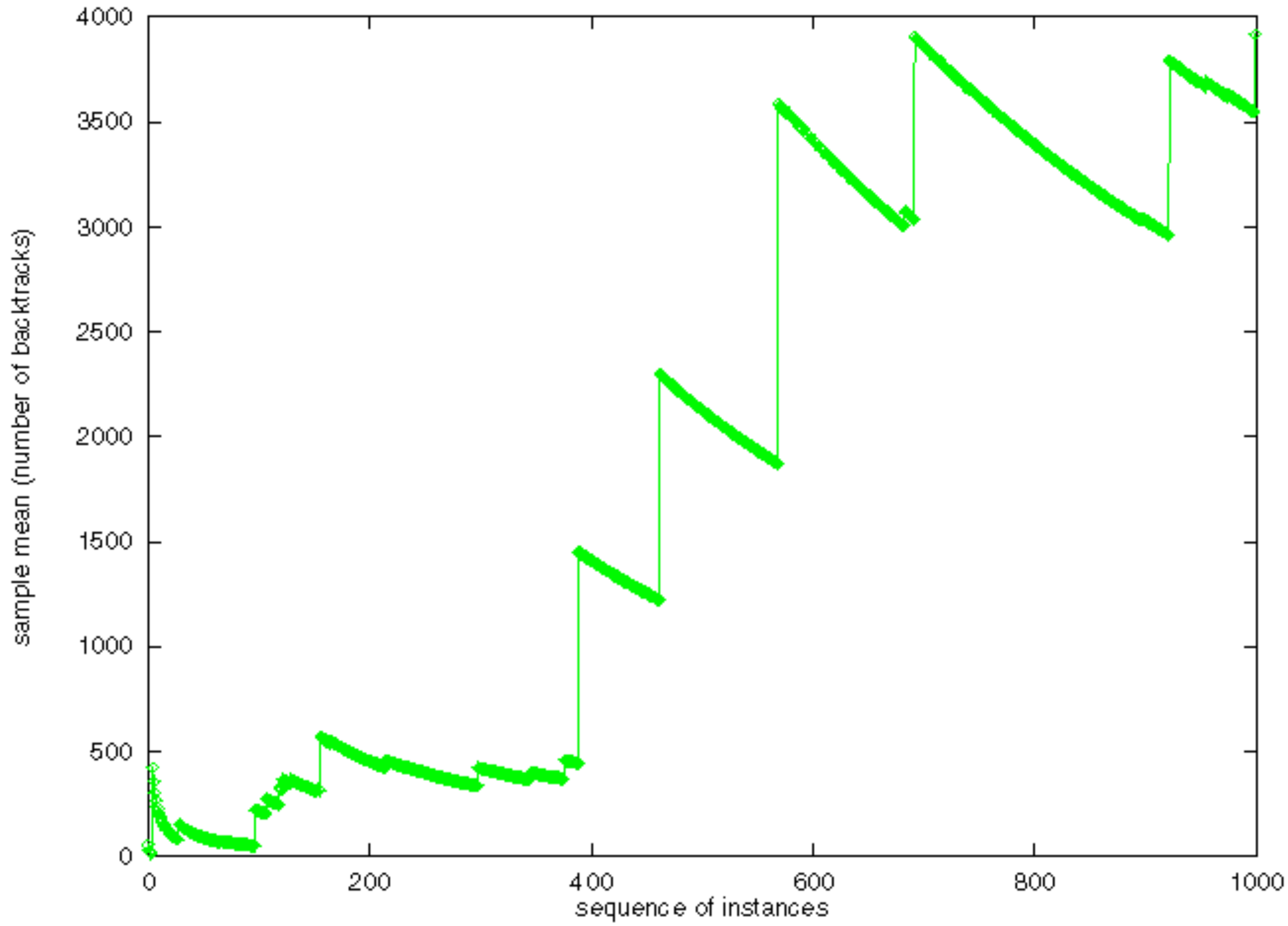
We’ll put variability / unpredictability to our advantage via randomization / averaging.

Cost Distributions

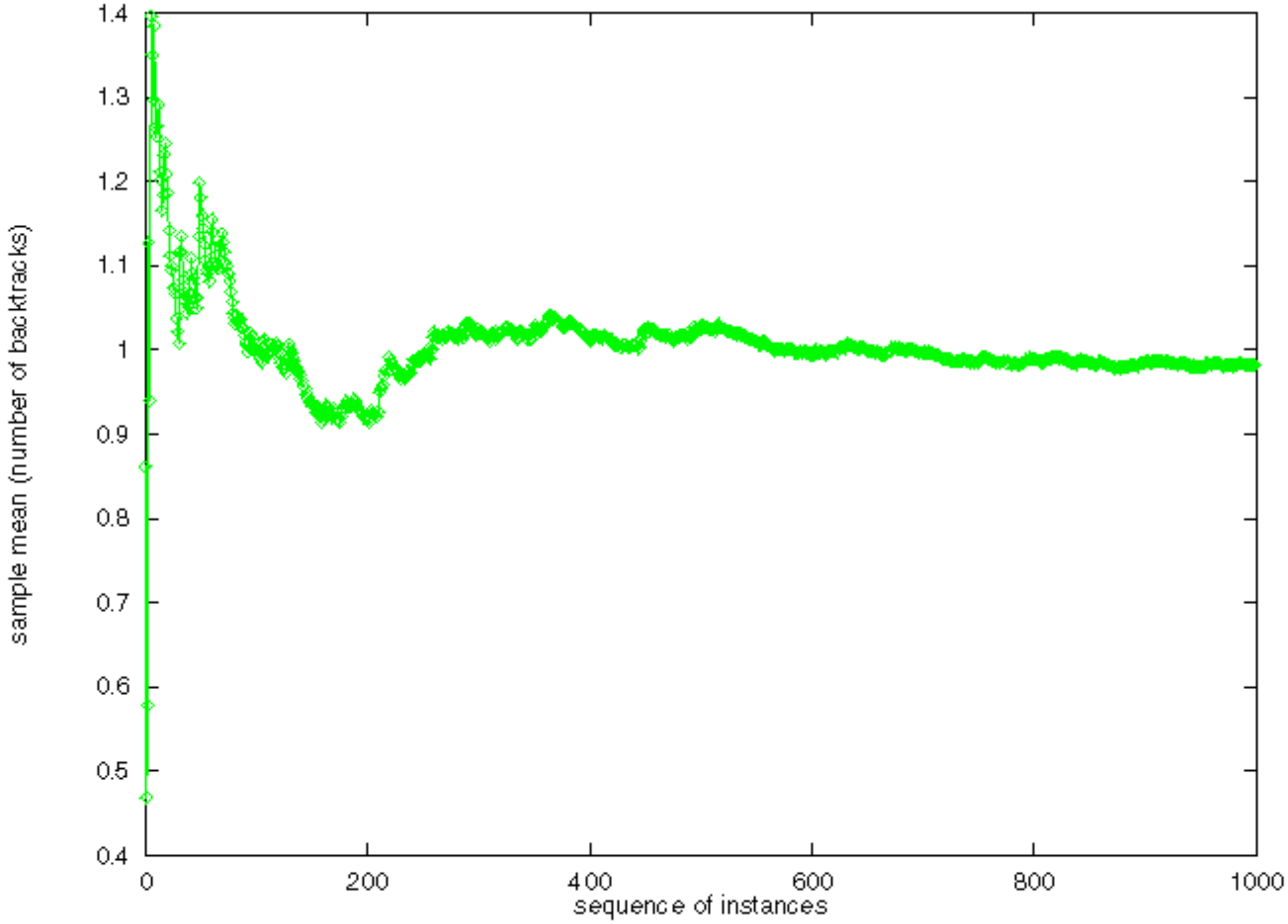
Backtrack-style search (e.g. Davis-Putnam) characterized by:

I Erratic behavior of mean.

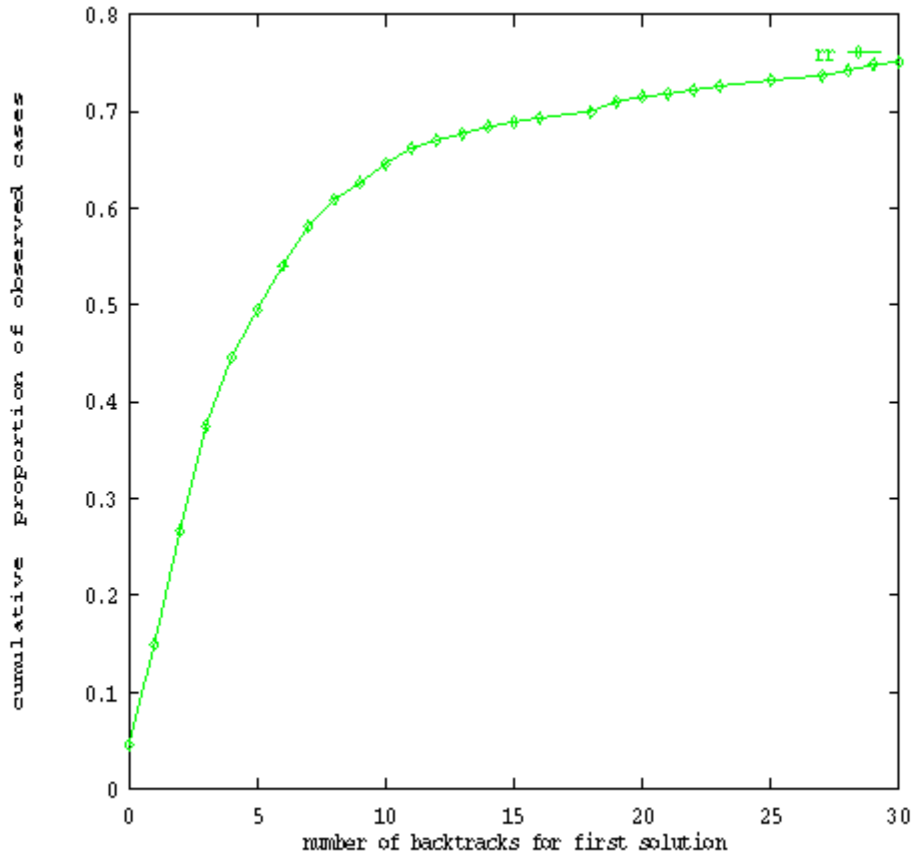
II Distributions have "heavy tails".

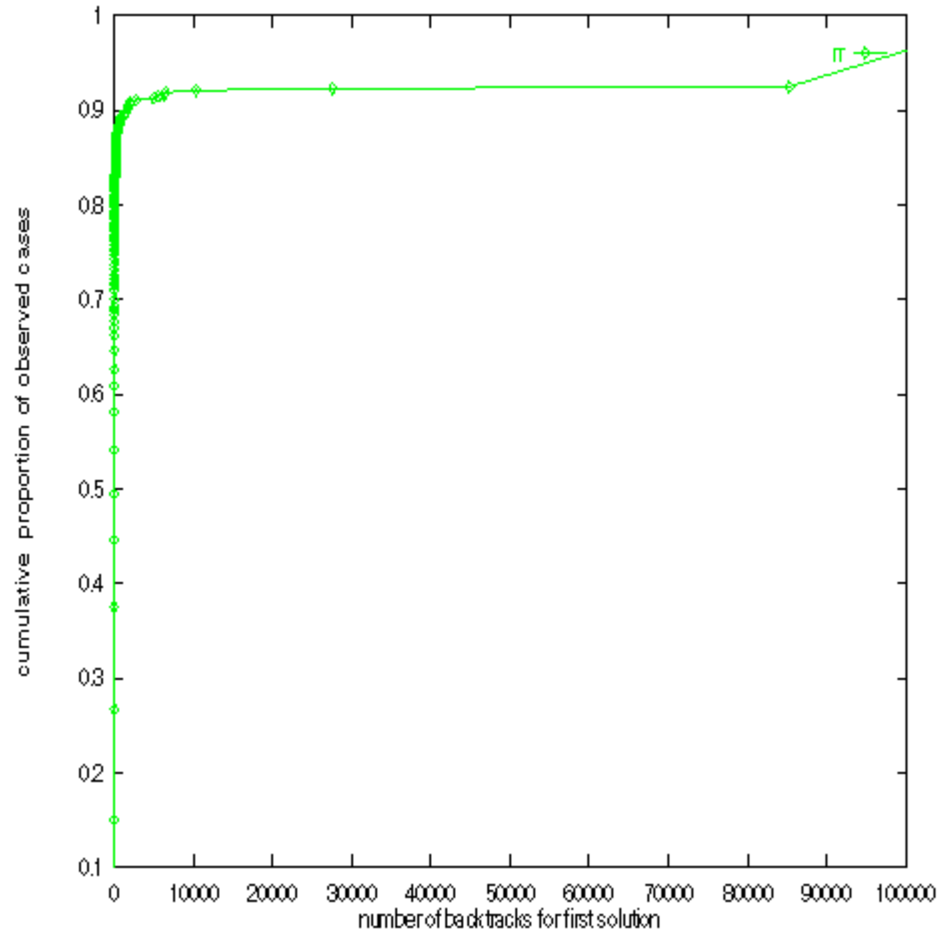


Erratic Mean Cost Behavior



Standard Mean Cost Behavior (Gamma)





Heavy-Tailed Behavior

Heavy-Tailed Distributions

… infinite variance … infinite mean

Introduced by Pareto in the 1920's

--- "probabilistic curiosity."

Mandelbrot established the use of heavy-tailed distributions to model real-world fractal phenomena.

Examples: stock-market, earth-quakes, weather,...

Decay of Distributions

Standard --- Exponential Decay

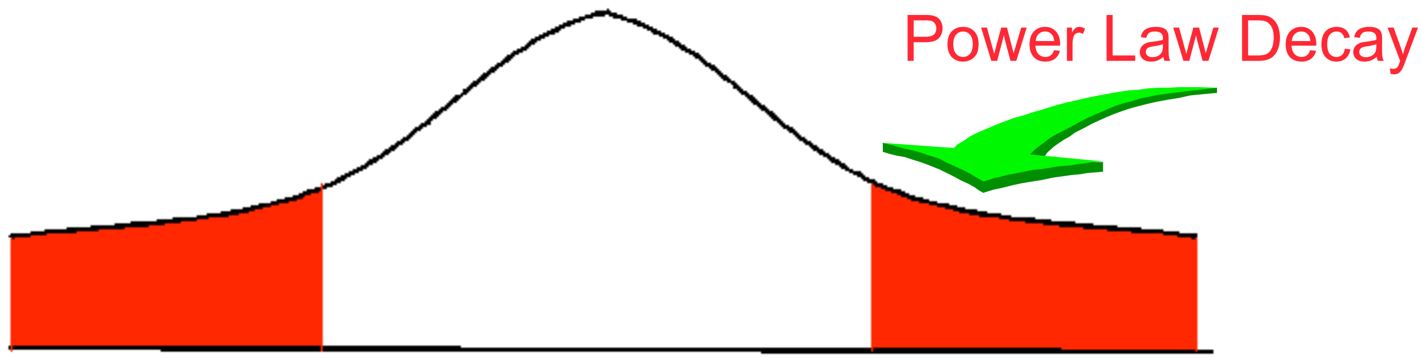
e.g. Normal:

[illegible]

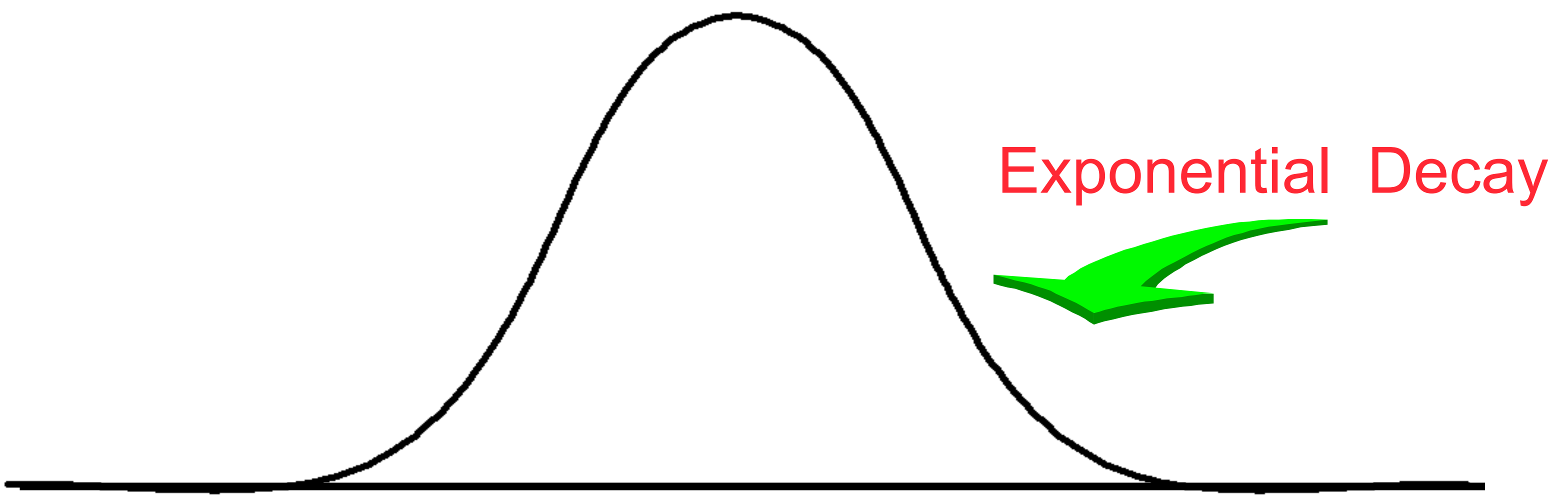
Heavy-Tailed --- Power Law Decay

e.g. Pareto-Levy:

[illegible]



HEAVY TAILED DISTRIBUTION
(infinite mean & variance)



Standard Distribution
(finite mean & variance)

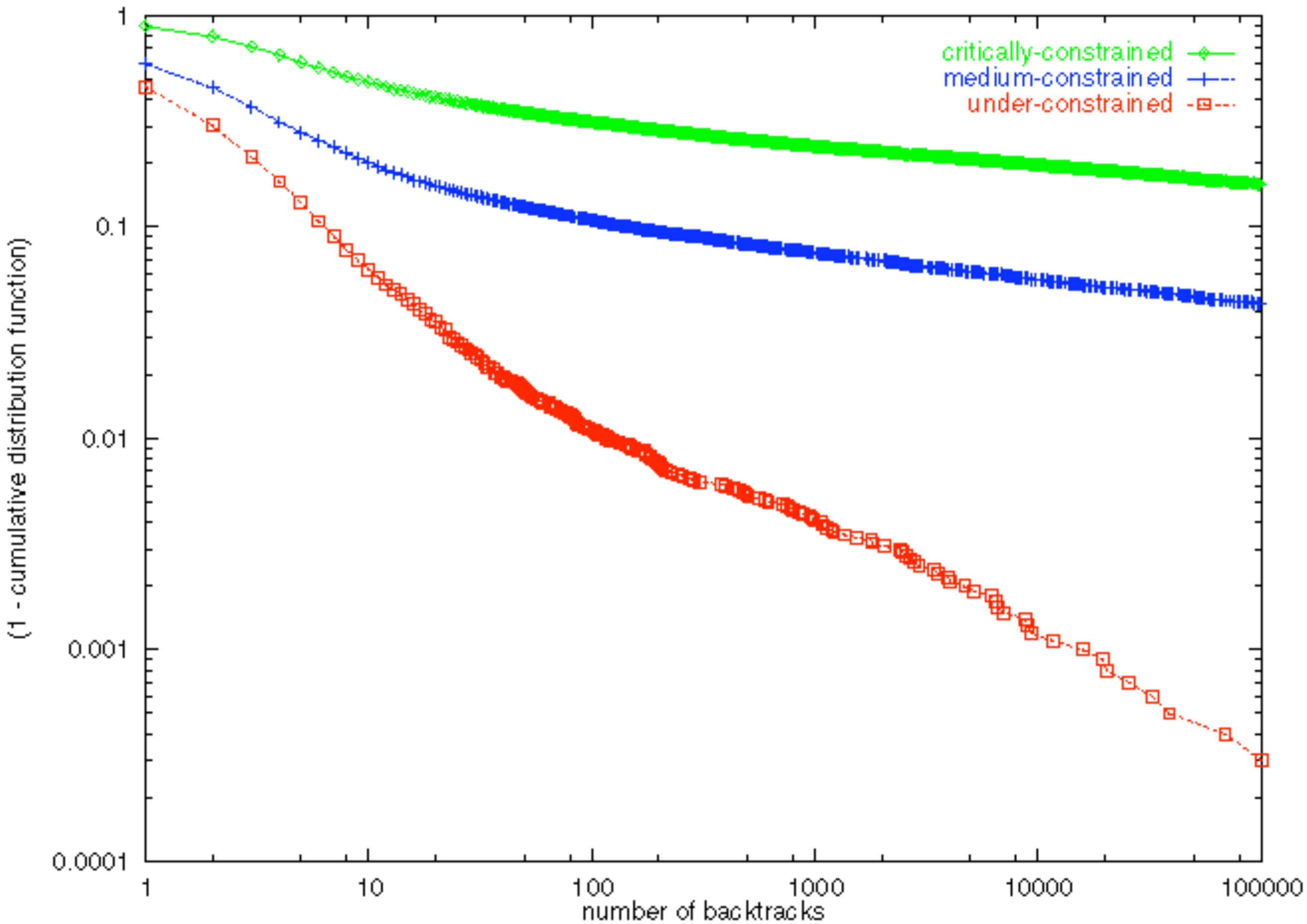
How to Check for “Heavy Tails”?

Log-Log plot of tail of distribution should be approximately linear.

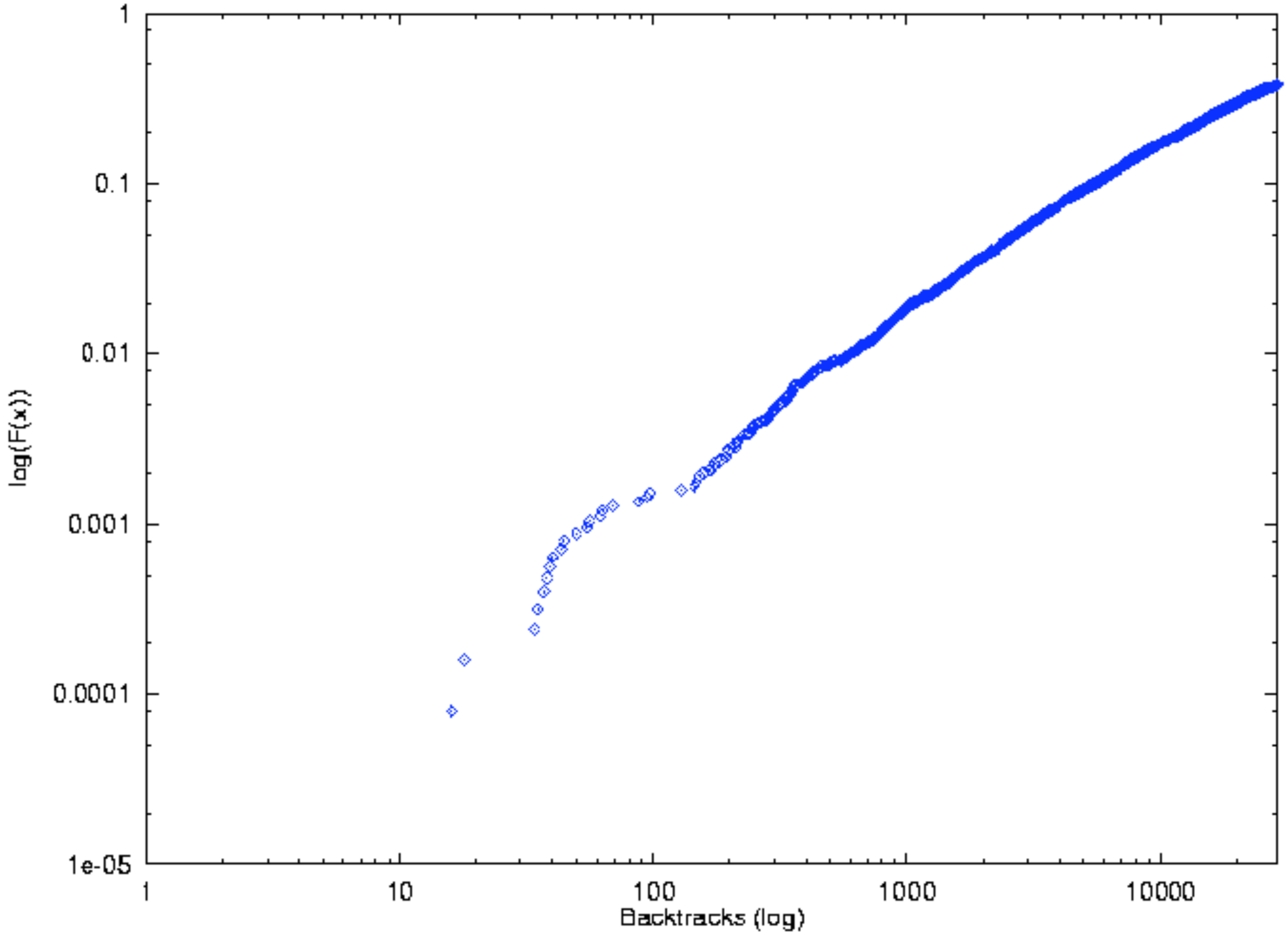
Slope gives value of [redacted]

[redacted] infinite mean and infinite variance

[redacted] infinite variance



Heavy-Tailed Behavior (log-log scale)


1
0.1
0.01
0.001
0.0001
1e-05
1
10
100
1000
10000
Backtracks (log)
log(F(x))

Heavy Tails

Bad scaling of systematic solvers can be caused by heavy tailed distributions

Deterministic algorithms get stuck on particular instances

but that same instance might be easy for a different deterministic algorithm!

Expected (mean) solution time increases without limit over large distributions

Randomized Restarts

Solution: randomize the systematic solver

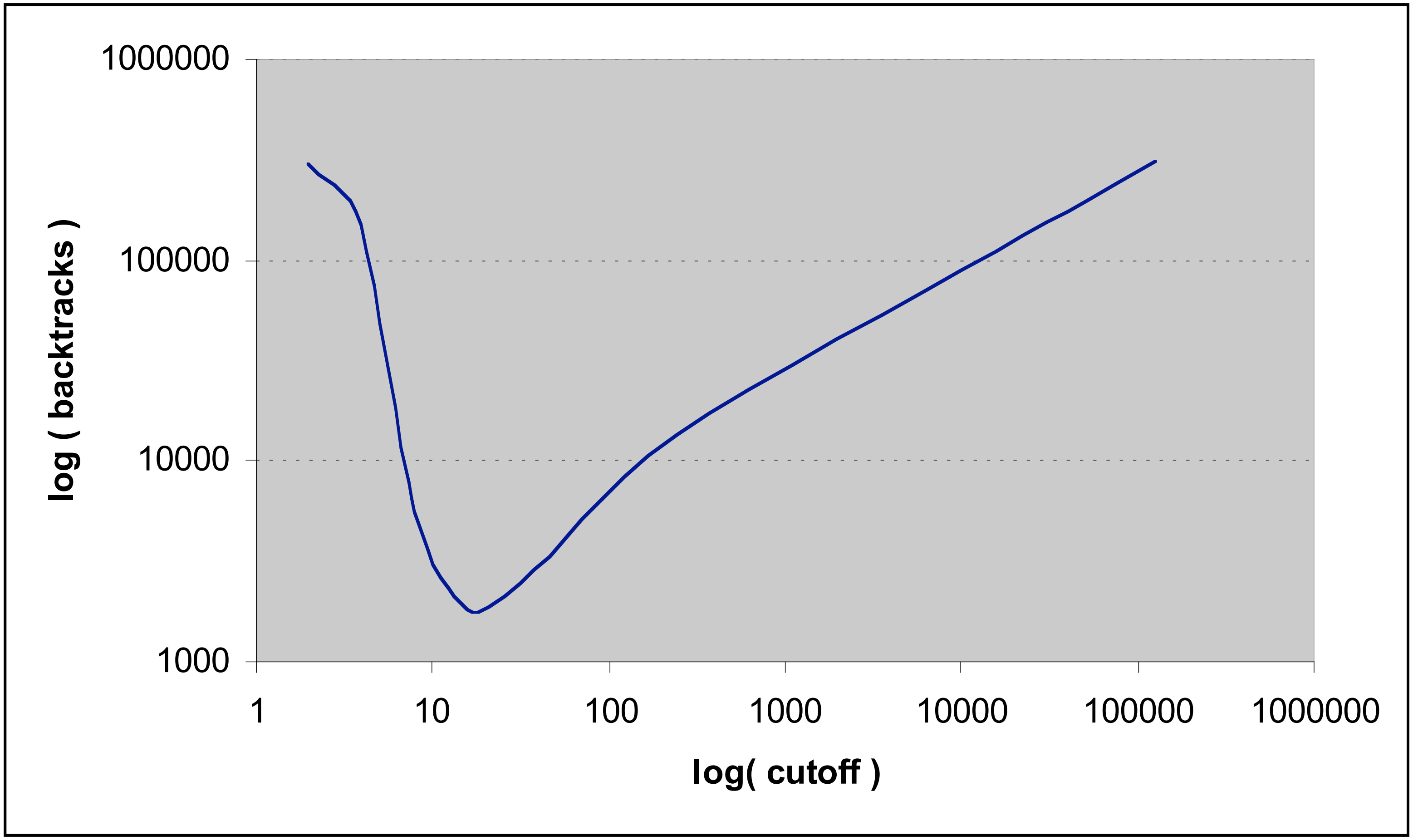
- Add noise to the heuristic branching (variable choice) function
- Cutoff and restart search after a fixed number of backtracks

Provably Eliminates heavy tails

In practice: rapid restarts with low cutoff can dramatically improve performance

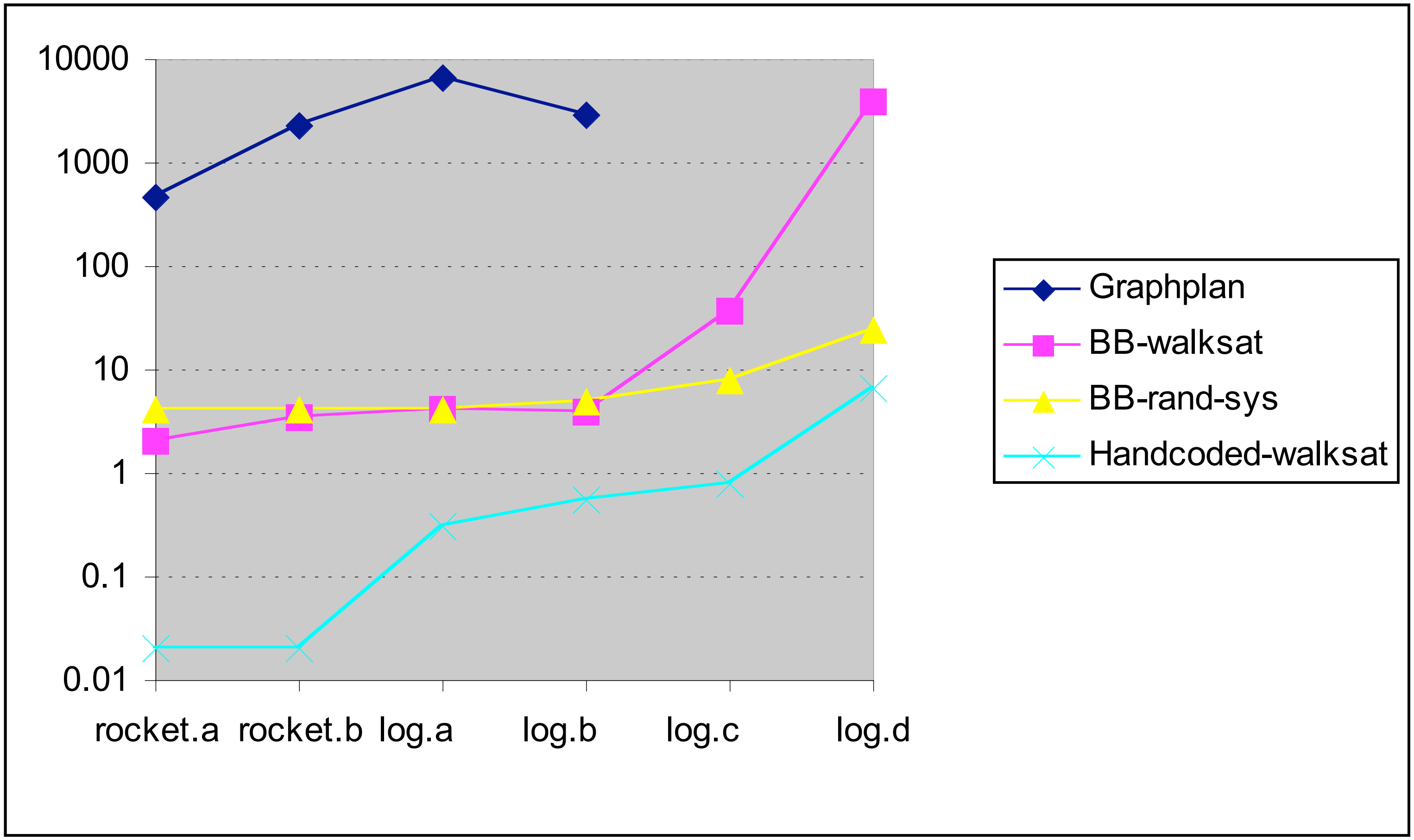
(Gomes and Selman 1997, 1998)

Rapid Restart on LOG.D



Note Log Scale: Exponential speedup!

SATPLAN Results



Overall insight:

Randomized tie-breaking with rapid restarts gives powerful bactrack-style search strategy.
(sequential / interleaved / parallel)

Related analysis: Luby & Zuckerman 1993; Alt & Karp 1996.

Heavy-Tailed Distributions in Other Domains

Quasigroup Completion Problem

Graph Coloring

Logistic Planning

Circuit Synthesis

Gomes, Selman, and Crato 1997 - Proc. CP97;
Gomes, Selman, McAloon, and Tretkoff 1998 - Proc AIPS98;
Gomes, Kautz, and Selman 1998 - Proc. AAAI98.

Sample Results Random Restarts

	Deterministic	R^3
Logistics Planning	108 mins.	95 sec.
Scheduling 14	411 sec	250 sec
Scheduling 16	---(*)	1.4 hours
Scheduling 18	---(*)	~18 hrs
Circuit Synthesis 1	---(*)	165sec.
Circuit Synthesis 2	---(*)	17min.

(*) not found after 2 days

Gomes, Kautz, Selman 1998

SAT Solvers: Themes, cont.

Randomization (as discussed)

Hybrid solvers --- Algorithm Portfolios

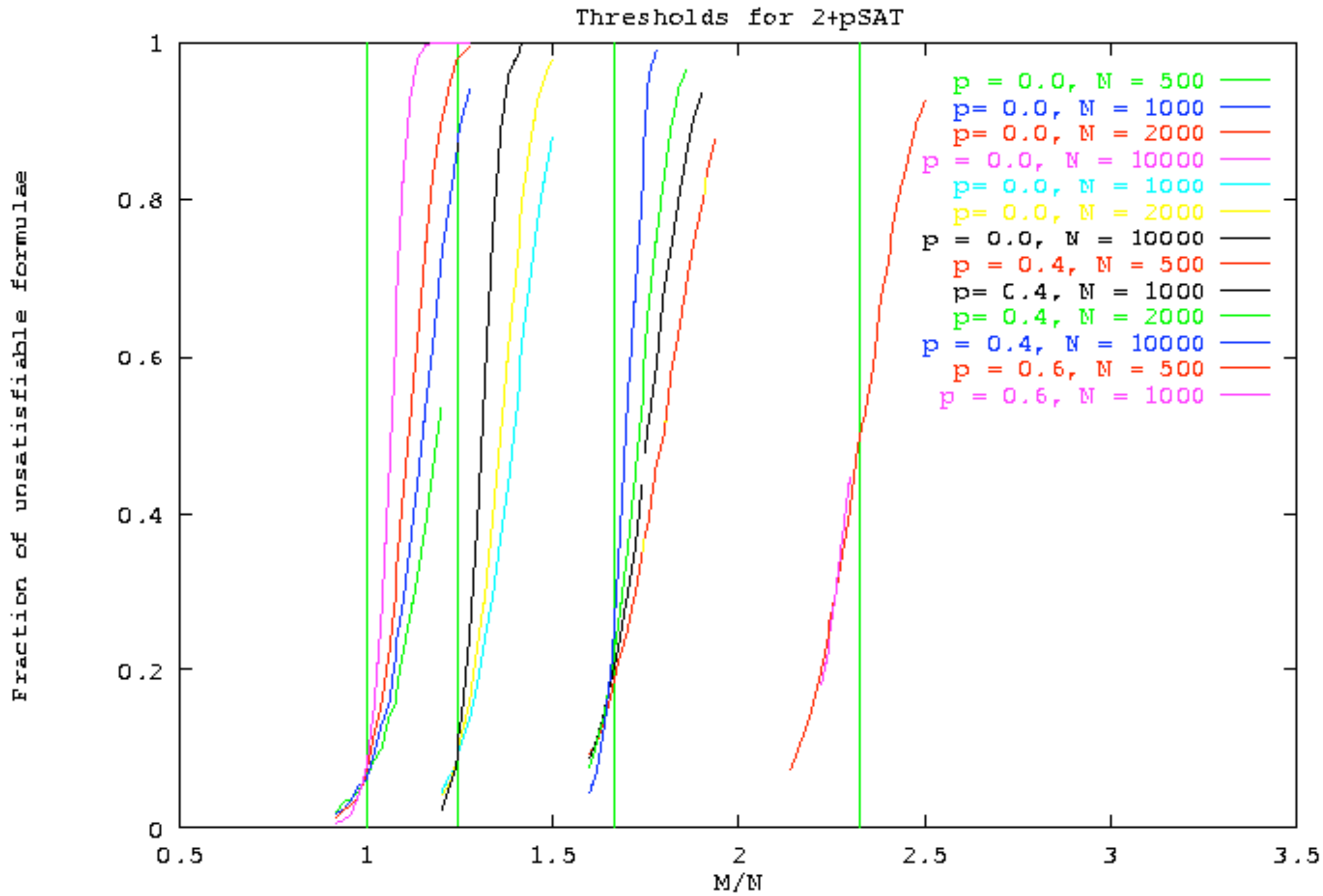
(Hogg & Hubermann 1997; Gomes & Selman 1997)

Using LP relaxations (Warners & van Maaren 1998)

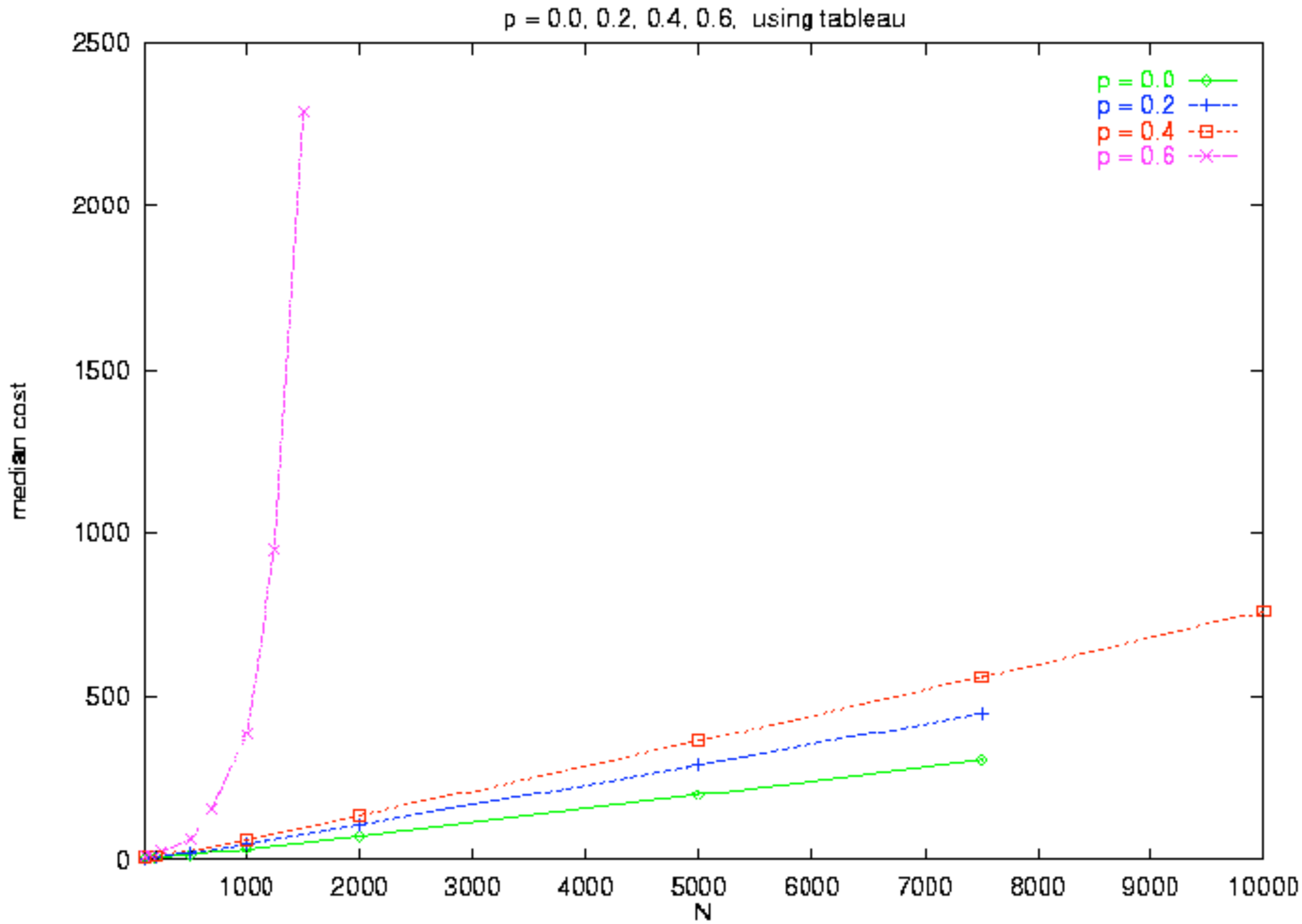
Between 2SAT / 3SAT:

Mixture can behave as pure 2SAT!

(Kirkpatrick, Selman, et al. 1996 / 1998)



Thresholds for 2+pSAT
Fraction of unsatisfiable formulae
M/N
0
0.2
0.4
0.6
0.8
1
0.5
1
1.5
2
2.5
3
3.5
p = 0.0, N = 500
p= 0.0, N = 1000
p= 0.0, N = 2000
p = 0.0, N = 10000
p= 0.0, N = 1000
p= 0.0, N = 2000
p = 0.0, N = 10000
p = 0.4, N = 500
p= C.4, N = 1000
p= 0.4, N = 2000
p = 0.4, N = 10000
p = 0.6, N = 500
p = 0.6, N = 1000


p = 0.0, 0.2, 0.4, 0.6, using tableau
median cost
N
p = 0.0
p = 0.2
p = 0.4
p = 0.6
0
500
1000
1500
2000
2500
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

SAT Solvers: Recent Theory

Minimal size of search tree

(Beame, Karp, et al. 1998)

Better worst-case: less than O(2^n)

backtrack style: O(2^(0.387n))

(Schiermeyer 1997; Paturi, et al. 1998)

local search: O(2^(c.n)) with c < 1

(Hirsch, 1998)

IV. Current Themes in Encodings

Add Declarative Domain Knowledge

Efficient representations and (randomized) SAT engines extend the range of domain-independent planning

Ways for further improvement:

- Better general search algorithms
- Incorporate (more) domain dependent knowledge

Kinds of Knowledge

* **About domain itself**

  a truck is only in one location

  airplanes are always at some airport

* **About good plans**

  do not remove a package from its destination location

  do not unload a package and immediate load it again

X **About how to search**

  plan air routes before land routes

  work on hardest goals first

Expressing Knowledge

Such information is traditionally incorporated in the planning algorithm itself

or in a special programming language

Instead: use additional declarative axioms

- Problem instance: operator axioms + initial and goal axioms + heuristic axioms
- Domain knowledge ≈ constraints on search and solution spaces
- Independent of any search engine strategy

Logical Status of Heuristics

1. **Entailed by operator axioms: conflicts and derived effects**

   fly(plane,d1,i) and fly(plane,d2,i) conflict

2. **Entailed by operators + initial state axioms: state invariants**

   a truck is at only one location

3. **Entailed by operators + initial + goal + length: optimality conditions**

   do not return a package to a location

4. **New constraints on problem instance: simplifying assumptions**

   Once a truck is loaded, it should immediately move

Axiomatic Form

Invariant: **A truck is at only one location**

$$at(truck,loc1,i) \ \& \ loc1 \neq loc2 \supset \neg\, at(truck,loc2,i)$$

Optimality: **Do not return a package to a location**

$$at(pkg,loc,i) \ \& \ \neg\, at(pkg,loc,i+1) \ \& \ i<j \supset \neg\, at(pkg,loc,j)$$

Simplifying: **Once a truck is loaded, it should immediately move**

$$\neg\, in(pkg,truck,i) \ \& \ in(pkg,truck,i+1) \ \& \ at(truck,loc,i+1) \supset \neg\, at(truck,loc,i+2)$$

Questions

Does it work?

- Additional axioms might just blow up instance with redundant information

Is effect independent of search engine?

Can we predict the most useful level of heuristic axioms?

- What is relation of difficulty to problem size?

Experiment: Logistics

h1: Optimality conditions

Once a package leaves a location, it never returns

h2, h3: Simplifying assumptions

A package is never in any city other than its origin or destination cities

- **rules out solutions where packages are transferred between airplanes in an intermediate city**

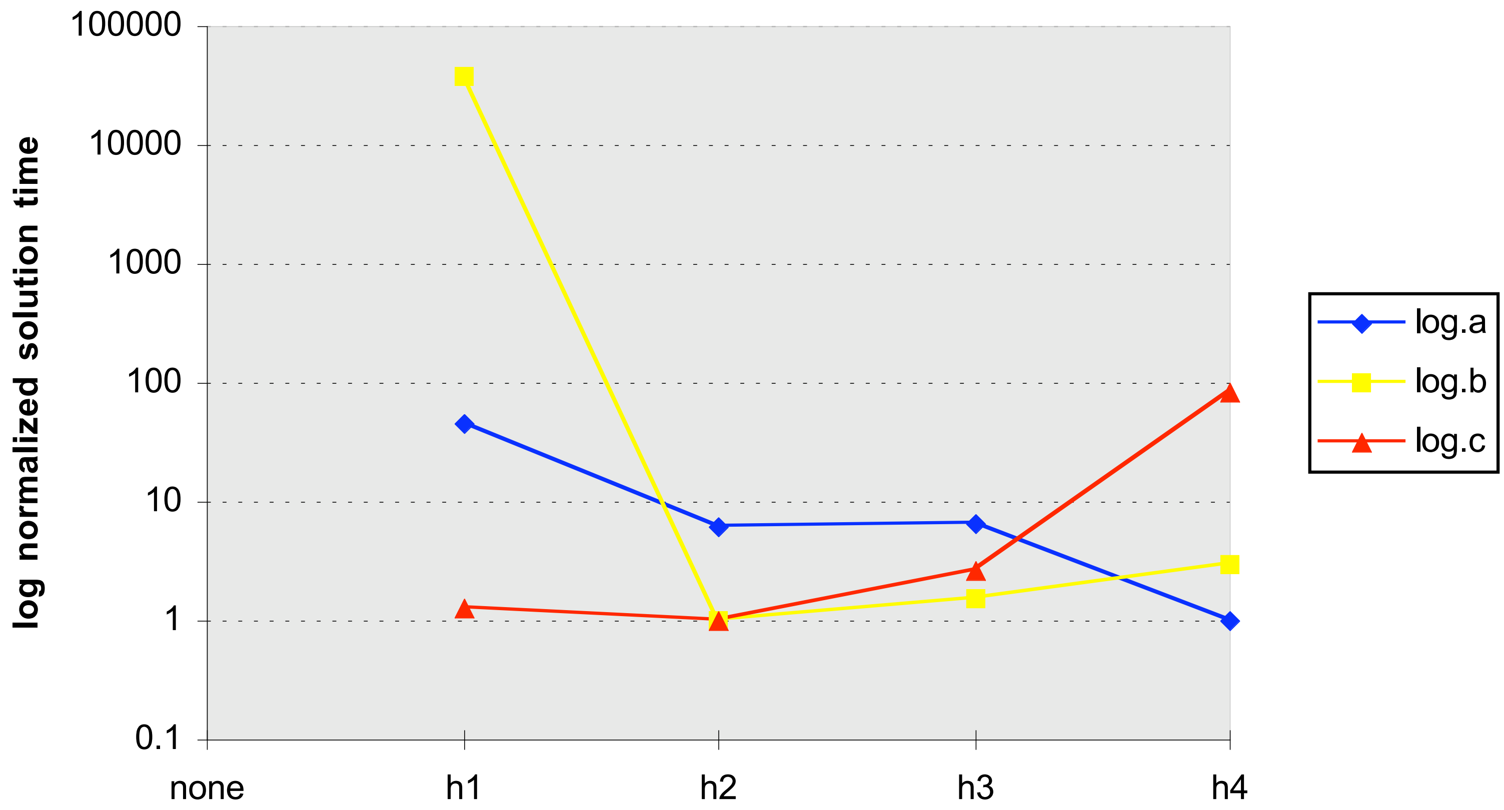
Once a vehicle is loaded, it should immediately move

- **rules out solutions where vehicles are loaded incrementally**

h4: More optimality conditions

A package never leaves its destination city

ntab solution of logistics



Answers

Does it work?

YES, 10 to 100+ times speedup

Is effect independent of search engine?

YES, same heuristics best for systematic and stochastic engines --- but needs more investigation

Can we predict the most useful level of heuristic axioms?

USUALLY point at which problem size is minimized after simplification by unit propagation (40% - 70% reduction)

How to Generate Control Knowledge --- Automatically

Polytime preprocessing

Try to add "obvious" inferences (McAllester, Crawford)

Compilation

Fix operators and initial or goal state, generate tractable equivalent theory (Kautz & Selman)

Learning strategies (Minton, Kambhampati, Etzioni, Weld, Smith)

Use automatic type inference to derive invariants.

(Fox & Long --- STAN system 1998; Rintanen 1998;
Koehler & Nebel --- IPP system 1998)

Encodings: Themes cont.

Add declarative control knowledge (as discussed)

Robustness

- Small change in original formulation, small change in encoding.

Add numeric information / “soft constraints”

- Weighted MAXSAT?

More compact encodings.

- E.g. causal.

Conclusions

Discussed current state-of-the-art in propositional reasoning and search.

Shift to 10,000+ variables and 10^6 clauses has opened up new applications.

Methodology: *Find compact SAT encoding;*
Use off-the-shelf SAT Solver.
Analogous to LP and MIP approaches.

Conclusions, cont.

Example: AI planning / SATPLAN system

One order of magnitude improvement (last 3yrs):

10 step to 200 step plans

Need two more:

up to 20,000 step ...

Discussed themes in SAT Sovers / Encodings

Heavy-tails / Randomization / Declarative domain knowledge