Name: ————————————-

Practice Exam 1
CS 591Q/791V - Pattern Recognition
Total points: 50
Posted on: April 6, 2010

Please read this first:
- **This is an in-class, closed-book exam consisting of 4 questions.** ☺
- **In order for your exam to be graded you will have to turn in your solutions by 12:30pm.**
- **You are not permitted to engage in *any* kind of discussion during the exam.**
- **If a question seems ambiguous, please state your assumption and proceed to solve it.**
- **An act of academic dishonesty will fetch you a 0 in the exam.** ☹

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1. [10 points] What are the three sources of error in a Bayes classification system? Explain with an example.

2. [15 points] Suppose we have three classes (ω_1, ω_2, ω_3) with $P(\omega_1) = 0.5$, $P(\omega_2) = 0.25$ and $P(\omega_3) = 0.25$ and the following distributions

$$p(x|\omega_1) \sim N(0,1); \quad p(x|\omega_2) \sim N(0.5,1); \quad p(x|\omega_3) \sim N(1,1)$$

and that we sample the following four points: $x = 0.6, 0.1, 0.9, 1.1$. Calculate explicitly the probability that the sequence actually came from $\omega_1, \omega_3, \omega_3, \omega_2$. Be careful to consider normalization.

3. [10 points] Consider a random variable x having the following distribution:

$$p(x|\theta) \quad = \theta^x (1-\theta)^{1-x}.$$

Suppose that n samples $(x_1, x_2, \ldots, x_n)$ are drawn independently according to $p(x|\theta)$. What is the maximum likelihood estimate of θ?

4. [15 points] Let $p(x|\omega_1) \sim N(\mu_1, \sigma_1^2)$ and $p(x|\omega_2) \sim N(\mu_2, \sigma_2^2)$, where the density parameters for both categories are unknown. Assume $P(\omega_1) = P(\omega_2) = 0.5$. Suppose the following training sets D_1 and D_2 are available for classes ω_1 and ω_2, respectively:

$$D_1 = \{0.67, 1.19, -1.20, -0.02, -0.16\},$$
$$D_2 = \{1.00, 0.55, 2.55, -1.65, 1.61\}.$$

   (a) Find the maximum likelihood estimates for the four unknown parameters based on the training sets D_1 and D_2.

   (b) What is the Bayes decision rule using the estimated values for the parameters? According to the Bayes decision rule, to which class would you assign the test pattern $x = 0.5$?

   (c) What is the Bayes error rate using the estimated values for the parameters?
