Homework 1

CS 591Q/791V - Pattern Recognition

Instructor: Dr. Arun Ross

Due Date: Feb 11, 2010

Total Points: 70

Note: You are permitted to discuss the following questions with others in the class. However, you *must* write up your *own* solutions to these questions. Any indication to the contrary will be considered an act of academic dishonesty.

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1. Chapter 1 of the book introduces an example of a pattern classification system that distinguishes sea-bass from salmon. Present a different example of a pattern classification system which may be of some commercial value.

   (a) [15 points] Briefly describe the (i) pattern classes, (ii) sensor employed, (iii) preprocessing and segmentation routines, and (iv) set of features used to distinguish the classes. Draw a schematic diagram illustrating the various stages of your pattern classification system.

   (b) [5 points] Why would you be interested in automating this classification problem?

   (c) [5 points] How will training be accomplished in your pattern classification system?

2. Consider the problem of classifying two-dimensional patterns of the form $\boldsymbol{x} = (x_1, x_2)^t$ into one of two categories, ω_1 or ω_2. Using the labeled patterns presented in this data set[1], do the following.

   (a) [10 points] Plot the two-dimensional patterns in a graph. Use markers to distinguish the patterns according to their class labels. Suppose you have the following decision rule to classify an arbitrary pattern $\boldsymbol{x} = (x_1, x_2)^t$:

   If $x_1 + x_2 - 15 < 0$, $\boldsymbol{x} \in \omega_1$ else $\boldsymbol{x} \in \omega_2$.

   In the same graph, plot the decision boundary corresponding to this rule. Also, report the confusion matrix and the error rate when this decision rule is used to classify the patterns in the given data set.

   (b) [10 points] Repeat the above after modifying the decision rule as follows:

   If $x_1 + x_2 - 12 < 0$, $\boldsymbol{x} \in \omega_1$ else $\boldsymbol{x} \in \omega_2$.

[1] The text file has 3 columns. The first two columns correspond to the feature vector of a pattern and the third column corresponds to its class label.

3. Consider a class-conditional density that has the following form for class ω_i:

$$p(x|\omega_i) = Ke^{-|x-a_i|/b_1}.$$

where $b_i > 0$.

(a) [10 points] Determine the value of K in terms of a_i and b_i.

(b) [5 points] For a two-class problem, assume that $a_1 = 0$, $b_1 = 1$, $a_2 = 1$, and $b_2 = 2$. Sketch the graph of the likelihood ratio $p(x|\omega_1)/p(x|\omega_2)$.

4. [10 points] Consider a 1-dimensional classification problem involving two categories ω_1 and ω_2 such that $P(\omega_1) = 2/3$ and $P(\omega_2) = 1/3$. Assume that the classification process can result in one of three actions:

α_1 - choose ω_1;
α_2 - choose ω_2;
α_3 - do not classify.

Consider the following loss function, λ:

$\lambda(\alpha_1|\omega_1) = \lambda(\alpha_2|\omega_2) = 0$;
$\lambda(\alpha_2|\omega_1) = \lambda(\alpha_1|\omega_2) = 1$;
$\lambda(\alpha_3|\omega_1) = \lambda(\alpha_3|\omega_2) = 1/4$.

For a given feature x, assume that $p(x|\omega_1) = \frac{2-x}{2}$ and $p(x|\omega_2) = 1/2$, $0 \leq x \leq 2$.

What is the Bayes minimum risk rule for classifying an input feature value x?