

• a)

$$\begin{aligned}
\hat{y}_o &= x'_o \hat{\beta} \\
&= x'_o (X'X)^{-1} X'Y \\
&= x'_o (X'X)^{-1} X' (X\beta + \epsilon) \\
&= x'_o \underbrace{(X'X)^{-1} X'X}_{I} \beta + x'_o (X'X)^{-1} X' \epsilon \\
&= x'_o \beta + \sum_{i=1}^n l_i(x_o) \epsilon_i
\end{aligned}$$

$$\begin{aligned}
\hat{y}_o &= x'_o \hat{\beta} + x'_o (\hat{\beta} - \beta) \\
&= x'_o \beta + x'_o \left((X'X)^{-1} X'Y - \beta \right) \\
&= x'_o \beta + x'_o \left((X'X)^{-1} X' (X\beta + \epsilon) - \beta \right) \\
&= x'_o \beta + x'_o \left(\underbrace{(X'X)^{-1} X'X}_{I} \beta + (X'X)^{-1} X' \epsilon - \beta \right) \\
&= x'_o \beta + x'_o \left(\beta + (X'X)^{-1} X' \epsilon - \beta \right) \\
&= x'_o \beta + x'_o (X'X)^{-1} X' \epsilon \\
&= x'_o \beta + \sum_{i=1}^m l_i(x_o) \epsilon_i
\end{aligned}$$

• b)

$$\begin{aligned}
EPE(x_o) &= E_{x_o} \left[E_{y_o/x_o} (y_o - f(x_o))^2 \right] \\
&= E_{\tau} \left[E_{y_o/x_o} (y_o - x'_o \beta)^2 \right] \\
&= E_{\tau} \left[E_{y_o/x_o} (y_o^2 - x'_o \beta \beta' x_o - 2y_o x_o \beta) \right] \\
&= E_{\tau} \left[V_{y_o/x_o}(y_o) + x'_o V_{y_o/x_o}(\beta) x_o - 2Cov(y_o, x_o \beta) \right], \text{ where } V(\beta) = (X'X)^{-1} \sigma^2 \\
&= E_{\tau} \left[\sigma^2 + x'_o (X'X)^{-1} x_o \sigma^2 - 0 \right] \\
&= \sigma^2 + E_{\tau} \left[x'_o (X'X)^{-1} x_o \right] \sigma^2
\end{aligned}$$

• c)

$$\begin{aligned}
EPE(x_o) &= \sigma^2 + E_{\tau} \left(x'_o (X'X)^{-1} x_o \right) \sigma^2 \\
&= \sigma^2 + E_{\tau} \left(x'_o Cov(X)^{-1} \frac{x_o}{N} \right) \sigma^2 \\
&= \sigma^2 + \frac{\sigma^2}{N} E_{\tau} \left(x'_o Cov(X)^{-1} x_o \right) \\
&= \sigma^2 + \frac{\sigma^2}{N} tr \left(x'_o Cov(X)^{-1} x_o \right) \\
&= \sigma^2 + \frac{\sigma^2}{N} tr \left(x_o x'_o Cov(X)^{-1} \right) \\
&= \sigma^2 + \frac{\sigma^2}{N} tr \left(Cov(X) Cov(X)^{-1} \right) \\
&= \sigma^2 + \frac{\sigma^2}{N} P \\
&= \sigma^2 + \left(\frac{P+N}{N} \right) \sigma^2
\end{aligned}$$